Year 12 Advanced Mathematics Sample Questions

Alternative Education Equivalency Assessments (AEEA)

ADVANCED MATHEMATICS

The following examples show the types of items in the test, but do not necessarily indicate the full range of items or test difficulty. For the Advanced Mathematics test, you may use a **silent, battery-operated, non-programmable scientific calculator** (not CAS/graphics calculator) and a ruler. See the Solutions pages for answers to these sample questions.

Formulae

The following formulae may be used in your calculations:

Quadratic Equations

$$If\ ax^2+bx+c=0\ then\ x=\frac{-b\pm\sqrt{(b^2-4ac)}}{2a}$$

Series

where *a* is the first term, L is the last, *d* is the common difference and *r* is the common ratio

Arithmetic $\quad a+(a+d)+(a+2d)+\ldots+(a+(n-1)d)=\frac{n}{2}(2a+(n-1)d)=\frac{n}{2}(a+L)$

Geometric $\quad a+ar+ar^2+\ldots+ar^{n-1}=\frac{a(1-r^n)}{1-r},\ r\neq 1$

Space & Measurement

In any triangle *ABC*,

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$Area=\frac{1}{2}ab\sin C$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\cos A=\frac{b^2+c^2-a^2}{2bc}$$

Trapezium: Area = $\frac{1}{2}(a+b)\times$ height, where *a* and *b* are the lengths of the parallel sides

Prism: Volume = Area of base $\times$ height

Cylinder: Total surface area = $2\pi rh+2\pi r^2$ Volume = $\pi r^2\times h$

Pyramid: Volume = $\frac{1}{3}$ $\times$ area of base $\times$ height

Space & Measurement (cont'd)

Cone: Total surface area = $\pi r s + \pi r^2$, s is the slant height   Volume = $\frac{1}{3} \times \pi r^2 \times h$

Sphere: Total surface area = $4\pi r^2$ Volume = $\frac{4}{3}\pi r^3$

Volume of solids of revolution about the axes: $\int \pi y^2\, dx$ and $\int \pi x^2\, dy$

Rate: If $y' = ky$, then $y = Ae^{kx}$

Temperature conversion formula

Degrees Celsius to degrees Fahrenheit: $^{\circ}F = (^{\circ}C \times 1.8) + 32$

Theorem of Pythagoras

In any right-angled triangle: $c^2 = a^2 + b^2$

Index laws

For $a, b > 0$ and m, n real,

$a^m a^n = a^{m+n}$ $a^m b^m = (ab)^m$ $(a^m)^n = a^{mn}$

$a^{-m} = \frac{1}{a^m}$ $\frac{a^m}{a^n} = a^{m-n}$ $a^0 = 1$

For m an integer and n a positive integer $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

Calculus

	Function notation		Leibniz Notation	
	y	y'	y	y'
Product rule	$f(x)\, g(x)$	$f'(x)\, g(x) + f(x)\, g'(x)$	uv	$\frac{du}{dx} v + u \frac{dv}{dx}$
Quotient rule	$\frac{f(x)}{g(x)}$	$\frac{f'(x)\, g(x) - f(x)\, g'(x)}{(g(x))^2}$	$\frac{u}{v}$	$\frac{\frac{du}{dx} v - u \frac{dv}{dx}}{v^2}$
Chain rule	$f(g(x))$	$f'(g(x))\, g'(x)$	$y = f(u)$ and $u = g(x)$	$\frac{dy}{du} \times \frac{du}{dx}$

Fundamental Theorem of Calculus: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$ and $\int_a^b f'(x)\,dx = f(b) - f(a)$

Standard Derivatives

If $y = f(x) = x^n$, *then* $y' = \frac{dy}{dx} = f'(x) = nx^{n-1}$

If $y = f(x) = e^x$, *then* $\frac{dy}{dx} = f'(x) = e^x$ *If* $y = f(x) = \log_e x$ *then* $y' = \frac{dy}{dx} = f'(x) = \frac{1}{x}$

If $y = f(x) = \sin(ax)$, *then* $y' = \frac{dy}{dx} = f'(x) = a\cos(ax)$

If $y = f(x) = \cos(ax)$, *then* $y' = \frac{dy}{dx} = f'(x) = -a\sin(ax)$

Standard Integrals

$$\int x^n dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1, \text{ and } x \neq 0 \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0 \qquad \int e^{ax} dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax \; dx = \frac{1}{a}\sin ax, \quad a \neq 0 \qquad \int \sin ax \; dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

Probability laws

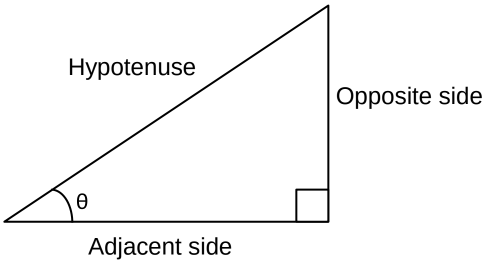
$P(A) + P(\bar{A}) = 1$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B/A) = P(B)P(A/B)$

$$\Pr(A/B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Trigonometry



In any right-angled triangle:

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

Growth, decay and interest formulae

- Simple growth or decay: $A = P(1 \pm ni)$
- Compound growth or decay: $A = P(1 \pm i)^n$

  Where:
  A = amount at the end of n years
  P = principal
  n = number of years
  r % = interest rate per year, $i = \frac{r}{100}$

Growth, decay and interest formulae (cont'd)

- Compound interest, where the interest is compounded t times per year:

  $$A = P(1 + \frac{i}{t})^{nt}$$

  Where:
  $t =$ number of interest periods per year

- Future value of an annuity: $F = \dfrac{x[(1+i)^n - 1]}{i}$ contributions at end of each period

  OR $F = \dfrac{x[(1+i)^n - 1] \times (1+i)}{i}$ contributions at beginning of each period

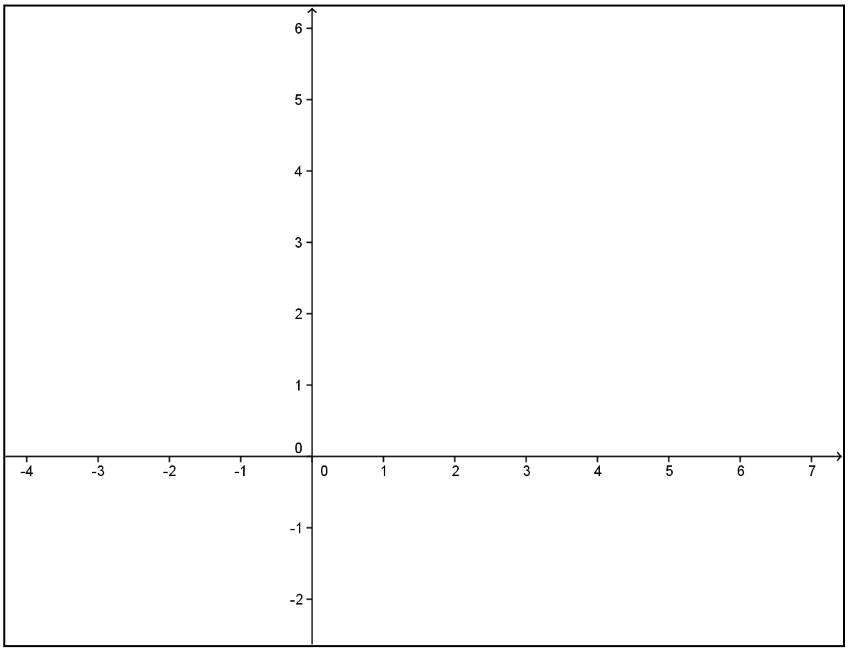
  Where:
  F = future value of annuity
  i = interest rate per compounding period, as a decimal fraction
  n = number of compounding periods

Real functions

Example 1 For the basic following functions: $f(x) = \frac{2x-1}{1+x}$ and $h(x) = 1 - 2x$ find the composite function , $f(h(x))$ in simplest terms: *(2 marks)*

Linear functions

Example 2 The line $2y + x = 4$ is reflected across the x axis. Sketch the original line and its reflection (clearly marking coordinates of any intercepts) then find the equation of the reflected line. *(3 marks)*

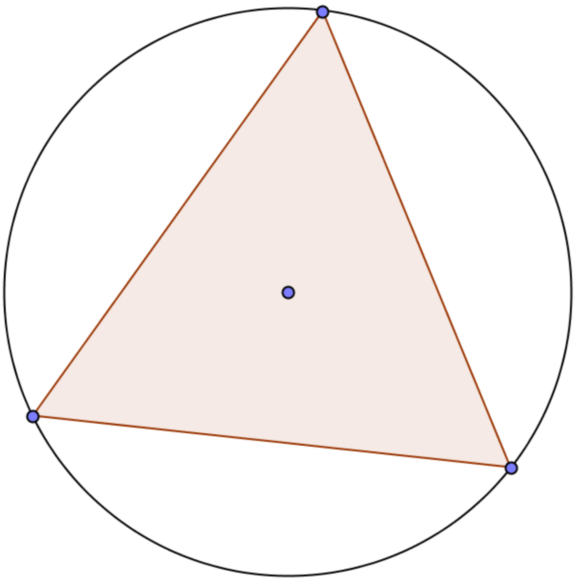


The quadratic polynomial and the parabola

Example 3 Find the coordinates of the turning point of the parabola $y = x^2 - 4x - 5$ *(2marks)*

Plane geometry – geometrical properties

Example 4 An equilateral triangle is inscribed in a circle of radius 3cm. Calculate the <u>unshaded area</u> as shown below (correct to 2 decimal places) *(3 marks)*



Tangent to a curve and derivative of a function

Example 5 Find the gradient of the curve $f(x) = 2e^{3x}$ at the point where $x = 1$ (correct to 2 decimal places) *(2 marks)*

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Coordinate methods in geometry

Example 6 The vertices of ΔABC are A(1,2), B(6,-1) and C(2,-2). Use your knowledge of the properties of a right angled triangle to show that ΔABC is a right angled triangle. *(2 marks)*

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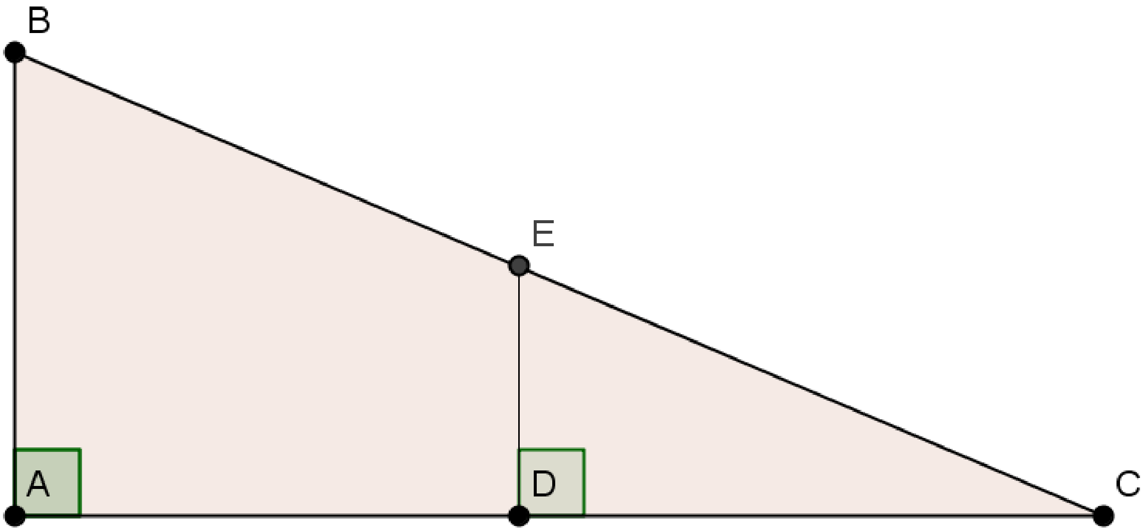
Applications of geometrical properties

Example 7 Given AB = 5 units, ED = 3 units and AD = 4 units, find the length of DC in the diagram below. *(2 marks)*



(diagram not drawn to scale)

Geometrical applications of differentiation

Example 8 Find the equation of the normal to the curve $y=(x-2)^2$ at the point where $x=3$. *(4 marks)*

Integration

Example 9 Find the exact value of $\int_0^2 \frac{1}{(2x+1)^2}\,dx$ *(2 marks)*

Trigonometric functions (including applications of trigonometric ratios)

Example 10 The function f(Ө) = 1 + sin2Ө is defined for Ө ϵ [0,2π]. Write down the maximum value of f(Ө) and the values of Ө for which it occurs. *(3 marks)*

Logarithmic and exponential functions

Example 11 An insect population grows according to the rule P = $2\log_e(t + 2)$ where P is the population, in millions, *t* years after the population was first estimated.

According to this rule:

a) What was the population when first estimated? *(1 marks)*

b) How long will it take for the population to reach 5 million? (correct to 2 decimal places) *(2 marks)*

Applications of calculus to the physical world

Example 12 With wind assistance, a balloon ascends at an acceleration of $2t$ m sec^{-2} (where t is the time in seconds after release). If the balloon is stationary until it is released from a height of 1 metre above ground level, how long will it take to reach a height of 100 metres? (correct to 2 decimal places) *(4 marks)*

Probability

Example 13 A tennis player wins 80% of her matches. To the nearest %, what is the probability she will win at least 4 of her next 5 matches? *(2 marks)*

Series and series applications

Example 14 A "not so wise" boss agreed to pay a worker \$1 on the 1st day, \$2 on the 2nd day, \$4 on the 3rd day and so on.

a) Show that this form of payment is a geometric sequence. *(1 marks)*

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b) How many dollars would the boss have to pay on the 20th day? *(2 marks)*

Year 12 Advanced Mathematics Sample Questions Solutions

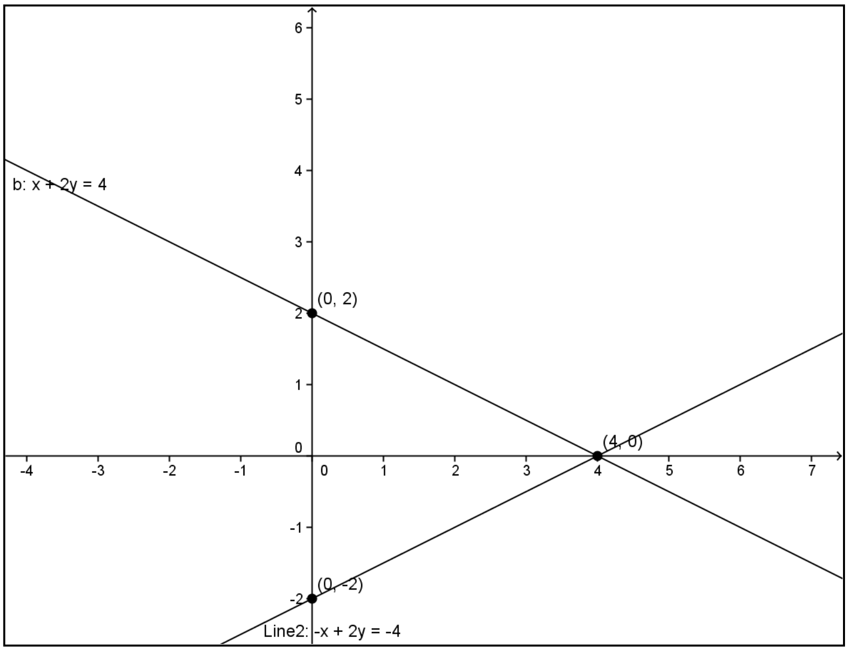
Example 1 solution

$$f(h(x)) = F(1-2x) = \frac{2(1-2x)-1}{1+(1-2x)} = \frac{1-4x}{2-2x}$$ ✓ ✓

Example 2 solution

New equation : $2y - x = -4$ or $x - 2y = 4$ or $-x + 2y = -4$ ✓

Both intercepts (or 2 points) must be shown on graphs



Example 3 solution

$y = (x - 2)^2 - 5 - 4$ **OR** $y = (x - 5)(x + 1)$

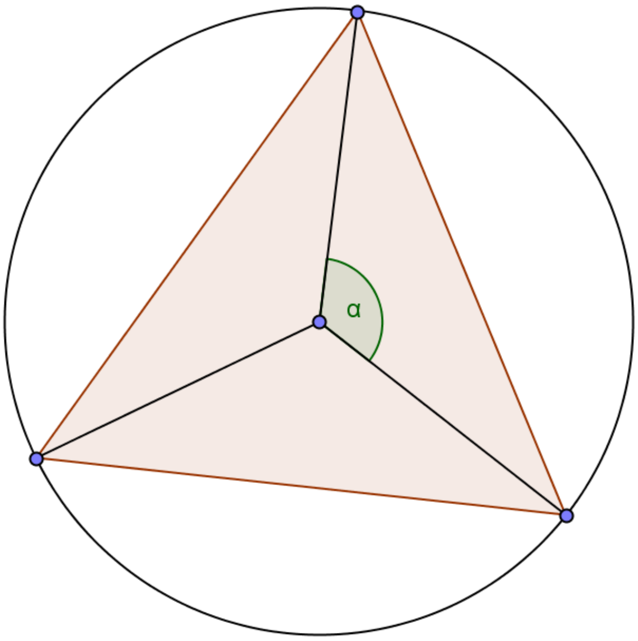
$y = (x - 2)^2 - 9$

TP at $x = \frac{5-1}{2} = 2$, $y = (2 - 5)(2 + 1) = -9$

$\therefore$ TP (2, -9) ✓ ✓

$\therefore$ TP (2, -9)

Example 4 solution



Area of 3 triangles side length 3cm, α = 120° = 3 x ½ (3)(3)sin120° = 11.6913
Area of circle $A = \pi r^2$ = π x 9 = 28.2743 ✓

Unshaded area = 28-2743 – 11.6913 = 16.58cm² ✓

Example 5 solution

f′(x) = $6e^{3x}$ ✓

f′(1) = $6e^{3}$ = 120.51 ✓

Example 6 solution

m(AC) = $\frac{-2-2}{2-1}$ = -4, m(BC) = $\frac{-2--1}{-4}$ = ¼ ✓

∴ -4 x ¼ = -1 so sides AC and BC are at right angles ΔABC is a right angled triangle ✓

OR

$AC^2 = (2-1)^2 + (-2-2)^2 = 17$ $AB^2 = (6-1)^2 + (-1-2)^2 = 34$ $BC^2 = (2-6)^2 + (-2- -1)^2 = 17$

∴ $BC^2 + AC^2 = AB^2$ so ΔABC must be a right angled triangle

Example 7 solution

Let DC = x

$\frac{5}{3} = \frac{4+x}{x}$ ✓

5x = 12 + 3x

∴ 2x = 12
∴ x = 6 ✓

Example 8 solution

$\frac{dy}{dx} = 2(x-2)$ ✓
∴ Gradient of tangent is 2(3 – 2) = 2 at x = 3 ✓

Gradient of the normal is:
$m = -\frac{1}{2}, \quad \frac{y-1}{x-3} = -\frac{1}{2},$ ✓
Equation of the normal $2y + x = 5$ ✓

Example 9 solution

$\int_0^2 \frac{1}{(2x+1)^2}\,dx = -\frac{1}{2}(2x+1)^{-1}\Big]_0^2$ ✓

$= -\left(\frac{1}{10}\right) - \left(-\frac{1}{2}\right)$

$= \frac{2}{5}$ ✓

Example 10 solution

Max value of 2
when:

$2\theta = \frac{\pi}{2}, \frac{\pi}{2} + 2\pi$

$\theta = \frac{\pi}{4}, \frac{\pi}{4} + \pi$

$\theta = \frac{\pi}{4}, \frac{5\pi}{4}$ ✓ ✓ (no working or sketch required - marks only for answers)

Example 11 solution

a) 1.39 million ✓

b) $5 = 2\log_e(t + 2)$ ✓

$e^{2.5} = t + 2$

$\therefore t = 10.18$ years ✓

Example 12 solution

Let x = height above ground level

$a = \ddot{x} = 2t$

$v = \dot{x} = \int 2t\,dt = t^2 + c$

At $t = 0$, $v = 0$ $\therefore c = 0$

$\therefore \dot{x} = t^2$ ✓

$x = \int t^2\,dt = \frac{1}{3}t^3 + c_1$

At $t = 0$, $x = 1$ $\therefore c_1 = 1$

$\therefore x = \frac{1}{3}t^3 + 1$ ✓

At $x = 100 = \frac{1}{3}t^3 + 1$ ✓

$\therefore t = \sqrt[3]{297} = 6.67$ seconds ✓

Example 13 solution

$\binom{5}{4}(0.8)^4(0.2)^1 + (0.8)^5$ ✓ (1 or both terms correct)

$= 0.4096 + 0.3277 = 74\%$ ✓

Example 14 solution

a) Common ratio = 2 i.e. $\frac{2}{1} = \frac{4}{2}$ ✓

✓

b) $a = 1$, $r = 2$, $t_{20} = ar^{19} = 1 \times 2^{19}$

= \$524,288 ✓

Notes

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